Rigid Triaxial Model for Depopulation of I = 3 Gamma Vibrational Band

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Resumen

Se emplea el modelo Davydov–Filippov para evaluar los branching ratios *B(E2) de las transiciones* $3^+ \rightarrow 2^+/4^+$, $3^+ \rightarrow 2^+/2^{+\prime}$ *y* $3^+ \rightarrow 4^+/2^{+\prime}$ *en la desexcitación de la banda vibracional gamma con* $I=3$ *de núcleos medianos y pesados par–pares deformados. Los resultados se comparan con valores experimentales conocidos. Se obtiene un excelente ajuste con un factor de dos para los núcleos cuyo parámetro de no axialidad* (γ) *se encuentra en el rango* $14^o < \gamma < 28^o$. *Esto justifica el uso de la aproximación adiabática para valores de energía mayores que 1 MeV y favorece la forma rígida de los núcleos con valores medios de los parámetros de forma* β *y* γ. *Clasifica además el nivel* $I=3$, $K^\pi=2$ *como miembro de la banda vibracional gamma originada por excitaciones colectivas, lo que contradice el punto de vista de Zawischa y otros, que pusieron en duda la naturaleza vibracional de los núcleos pares deformados.*

Descripción de la desexcitación de la banda vibracional gamma (I=3) por el modelo del rotor rígido triaxial

Abstract

The Davydov–Filippov Model has been employed to evaluate the B(E2) branching ratios $3^+ \rightarrow 2^+/4^+$, $3^+ \rightarrow 2^+/2^{+\prime}$ *and* $3^+ \rightarrow 4^+/2^{+\prime}$ *depopulating* $I=3$ *gamma vibrational band of the even–even deformed nuclei in medium and heavy mass region. The results are compared with the known experimental values. An excellent fit within a factor of two has been obtained for nuclei having non–axiality parameter* (γ) *in the range* $14^o < \gamma < 28^o$. *It supports the adiabatic approximation at the energy values more than 1 MeV and favours the rigid shape of the nucleus with mean values of shape parameters* β *and* γ. *It also establishes* $K^\pi = 2$, $I=3$ *level as a member of classical gamma vibrational band originating from collective excitations and, therefore, goes against the view point of Zawischa* et al, *who doubted the vibrational nature of even deformed nuclei.*

INTRODUCTION

The vibrational levels in even–even deformed nuclei can be regarded as originating from the two phonon states in the spherical nuclei. The nucleus is considered as an incompressible liquid drop with a sharp surface [1]. Davydov–Filippov (DF) [2] showed that the violation of axial symmetry generates new states with spin 2, 3, 4, ... while the rotational spectrum of axially symmetric nucleus remains almost unchanged. The shape of the nucleus changes from a prolate to an oblate ellipsoid if the deformation parameter (β) remains fixed while the non–axiality parameter (γ) varies from 0 to $\pi/3$. The value of $\gamma = 30^0$ corresponds to a shape between prolate and oblate ellipsoid of revolution. This axially asymmetric model has been found very successful [3–5] in explaining the rotational levels of the deformed even–even nuclei, the large observed electric quadrupole moments and the transition probabilities. Although this model has been very successful in describing the depopulation of I= 3, gamma vibrational band in some selected isotopes of chains of Sm, Ru and Pd nuclei [6–7], no attempt has been made till now to study the systematics of the E2 transitions from 3^+ level of gamma vibrational band according to this model. One of the reasons may be the possible break down of the adiabatic approximation [8] (i.e. fixed values of β and γ) at the energy of spin 3^+ level which exceeds 1 MeV. Later rigorous calculations according to Davydov–Rostovsky (DR) model [8] were done by Abecasis *et al* [19], who observed the equivalence of Rotation Vibration Model (RVM) and DR models for the description of transition ratios inspite of the discrepancies shown by both of them. They further observed that RVM gave satisfactory results in which DR model predicted unphysical situation. Toyama [10], adopting an asymmetric shape of a nucleus and on introducing an anharmonic term in the Hamiltonian, calculated the relative B(E2) values which also showed discrepancy with experimental values. The reason proposed by him was that 3^+ state may not be affected by the perturbation adopted and so the deviation of theoretical values from the experimental ones in the B(E2) transition ratio $3^+ \rightarrow 4^+ / 2^+$ remained large. Moreover he considered only the nuclei in heavy mass region ($8^0 < \gamma < 15^0$) which do not reflect the characteristics of asymmetric rotor. Zawischa *et al* [11] studied the low–lying and high–lying $K^\pi = 0^+, 2^+$ states for nuclei in the deformed rare earth region in the framework of the quasiparticle random phase approximation and interpreted high–lying $K^\pi = 0^+, 2^+$ resonances as the classical β and γ vibrations. Since at present [12–15] a lot of new data on B(E2) branching ratios are available, we thought it worthwhile to study the systematics of B(E2) branching ratios in the framework of DF model. Some available results of microscopic models are also given for comparison.

The present study is made to investigate the following possibilities:

The validity of adiabatic approximation above 1 MeV (1), i.e. the energy of 3^+ level of gamma vibrational band, since earlier [16] it was observed that the nucleus started getting rid of its rigid shape at $I^\pi = 6^+$ in the ground state rotational band when the energy exceeded 1 MeV (2). Is the concept of increase in the value of non–axiality parameter γ with the increase of spin I, employed in explaining B(E2) drop near and after back bending observed in some nuclei [17], applicable in the gamma band also? (3). The confirmation of viewpoint of Zawischa *et al* [11] for $K^\pi = 2^+$ level has not been found to be consistent with $K^\pi = 0^+$ level in the earlier work [18]. As the absolute B(E2) values for $K^\pi = 2^+$, I= 3 levels are yet to be measured for most of the nuclei, we have compared the available B(E2) branching ratios which provide a stringent test of a nuclear theory. The depopulation of I=2, $K^\pi = 2^+$ level had already been studied earlier [16] but a part of it is also presented here and compared with Zawischa's results to draw a meaningful inference. The value of non–axiality parameter (γ) has been calculated from the energy ratio of $2^{+'}$ and 2^+ levels and the same value has been used for the branching ratio calculations for I= 3^+ level. Non–axiality parameter (γ) was also calculated from the energy ratio $E3^+/E2^+$ to study its variation with the spin of the level. Branching ratios for some of the nuclei were also calculated with different values of γ to examine its influence. Although the reproduction of the level energies is the initial requirement to test any nuclear model, it has been observed that the nuclear models are not in general capable of predicting the level energies and B(E2) ratios simultaneously with the same accuracy. We have made rigorous calculations for the energy and branching ratios of 3^+ level according to DF model to see up to what extent Asymmetric Rotor Model (ARM) provides the energy fit together with branching ratios.

METHOD OF CALCULATION

Experimental B(E2) branching ratios B(E2; $3^+ \rightarrow 2^+/4^+$), B(E2; $3^+ \rightarrow 2^+/2^{+'}$), and B(E2; $3^+ \rightarrow 4^+/2^{+'}$) for transitions depopulating I= 3 gamma vibrational level are evaluated taking the gamma–ray energies and intensities for these transitions from Table of Isotopes [13]. The mixing ratio factor is applied for those transitions which have MI mixing and the internal conversion coefficient values used have been taken from reference [19].

The rigid triaxial model calculations are done using DF relations [2]. The value of γ has been obtained [2] from the ratio $s = E2^{+'}/E2^+$. The energy of 3^+ level and ARM dependent Q_0 have been evaluated using the expressions given in reference [2] and [5].

RESULTS AND DISCUSSION

It is well known that the ARM characteristics in deformed nuclei are well reflected when the nuclei have large value of asymmetric parameter (γ) [5]. Unfortunately, Zawischa *et al* compiled only those nuclei which have small values of γ. Even then it can be observed from table 1 that

DF results are comparable with those of Zawischa in general and are better in particular cases where Zawischa theory breaks down. Table 1 illustrates the energy ratio s($= E2^{+\prime}/E2^{+}$), non–axiality parameter γ, experimental, DF and Zawischa values of B(E2; $0^+ \rightarrow 2^{+\prime}$) for nuclei listed in reference [11]. The experimental and Zawischa values are adapted from reference [11]. Imposing Kumar's condition [20] (i.e. $0,5 <$ enhancement/hindrance factor $F < 2$) on both the models, we find that $^{172-176}$Yb nuclei keep themselves out of DF discipline (F=5); while Zawischa *et al* fail to accomodate ^{174}Hf ($F \simeq 200$) and ^{186}W ($F \simeq 4$). It is interesting to note that for ^{186}W, the factor F reduces to 2 in DF from a value of 4 in Zawischa, since it has moderate γ value ($= 15,8^0$). Systematic of B(E2; $0^+ \rightarrow 2^{+\prime}$) versus s is plotted in figure 1 and it is observed that the two theoretical values lie on both sides of experimental line. Therefore, it is inferred that even at low values of γ DF results are as good as that of Zawischa.

Table 2 lists B(E2; $3^+ \rightarrow 2^+/4^+$); B(E2; $3^+ \rightarrow 2^+/2^{+\prime}$), and B(E2; $3^+ \rightarrow 4^+/2^{+\prime}$) values in even–even deformed

Table 1

B(E2; $0^+ \rightarrow 2^{+\prime}$) values in e^2fm^4. The theoretical values which deviate from experiment by a factor of 2 are underlined

Nucleus	*s*	γ	*Exp.*	*Zawischa*	*DF*
^{152}Sm	8,90	13,25	1190(240)	627	1450
^{154}Gd	8,09	14,00	1300(500)	1058	1900
^{156}Gd	12,97	11,00	980	564	1550
^{158}Gd	14,93	10,3	1060	752	1450
^{160}Gd	13,13	11,0	1100(30)	890	1750
^{158}Dy	9,56	12,7	1640	737	2150
^{160}Dy	11,13	11,7	1050(80)	942	1950
^{162}Dy	11,01	12,0	1030(50)	1250	2100
^{164}Dy	3,144	21,75	1010(60)	1290	2200
^{164}Er	9,41	12,9	1800(500)	865	2550
^{166}Er	9,75	12,75	1400(60)	761	2300
^{168}Er	10,29	11,3	1300(50)	782	2250
^{170}Er	11,86	11,4	1000(60)	648	2100
^{172}Yb	18,61	9,5	300	198	<u>1500</u>
^{174}Yb	21,36	9,0	400	370	<u>1400</u>
^{176}Yb	15,35	10,25	600(150)	531	<u>1500</u>
^{174}Hf	13,48	10,8	1380(200)	<u>6,8</u>	<u>1550</u>
^{176}Hf	15,18	10,25	1240(50)	<u>386</u>	1800
^{178}Hf	12,61	11,3	1130(120)	<u>462</u>	1800
^{182}W	12.20	11,4	1240(60)	745	2150
^{184}W	8,12	13,8	1380(60)	<u>634</u>	2600
^{186}W	6,03	15,8	1390(40)	<u>381</u>	2850

nuclei. Some of the experimental values have been taken from reference [10] [12–14] and therest have been evaluated from measured energies and intensities. The theoretical values which deviate by more than a factor of five are underlined. An overall excellent fit is observed. However, for smaller values of γ ($< 10^0$) the situation is different which could be improved much if some enhancement is made in the values of γ accounting for the Bohr Mottelson Rotation Vibration Interaction Correction (BMRVIC), as reported earlier [7] for samarium isotopes. The ^{74}Ge nucleus ($\gamma = 29^0$) needs some reduction in the value of γ as suggested in reference [7]. The nuclei ^{108}Cd, ^{158}Gd, ^{230}Th show larger deviations from experimental values which may be due to the fact that the transitions taken may have larger MI mixing ratio than given in reference [13].

Figures 2–4 are plots of B(E2; $3^+ \rightarrow 2^+/4^+$); B(E2; $3^+ \rightarrow 2^{+}/2^{+\prime}$), and B(E2; $3^+ \rightarrow 4^+/2^{+\prime}$) as a function of s. It can be observed that the experimental and DF values nearly coincide and show the same trend up to $s = 8$ ($\gamma \simeq 14^0$), but little deviation starts as s exceeds 8. We can infer that for $2 < s < 5$ i.e. $28^0 > \gamma > 14^0$ the DF model gives an excellent fit both in quantity and quality. For $\gamma < 14^0$, only quantity is retained and quality can be brought back by enhancing the value of γ by 2^0 or 3^0 which may account for BMRVIC.

Table 3 shows B(E2; $3^+ \rightarrow 2^+/2^{+\prime}$) values for samarium isotopes. The microscopic model results are listed for comparison only. Experimental, Dynamic Pairing Plus Quadrupole (DPPQ) and Boson Expansion Model (BEM) values are taken from reference [14]. We have not included B(E2; $3^+ \rightarrow 2^+/2^{+\prime}$) experimental value for ^{152}Sm of reference [14] since Table of Isotopes and also the reference [21] quoted by Gupta [14] do not give such transition. BEM breaks down for ^{150}Sm as the enhancement factor F exceeds 9.

Table 4 shows B(E2; $3^+ \rightarrow 2^+/4^+$) values in respect of $^{98-104}$Ru and $^{102-110}$Pd nuclei. The DF results are as good as the microscopic model values for Ru and Pd nuclei.

Table 5 shows the values of γ_1 and γ_2 derived from the energy ratios $E2^{+\prime}/E2^+$ and $E3^+/E2^+$ respectively. We notice that there is in general very little change or almost no change in the value of non–axiality parameter γ. The vacant places are left where 3^+ level are not known. This observation excludes the possibility of variable–γ–approach [17] for describing the B(E2) ratios for the spin 3 transitions.

Table 6 present the calculations for the Ru and Pd isotopes for which two γ values are slightly different, but almost no change in B(E2) ratios is observed. This in turn supports the assumption of adiabatic approximation at this spin also.

Table 7 shows the theoretical and experimental energies of $E3^+$ level, and a good agreement in general is found. Although the analysis of level energies is not a very good probe for the nuclear shape as they are insensitive to softnees even then, the simultaneous excellency achieved by ARM in describing the level energies and the B(E2) branching ratios in respect of 3^+ level of gamma vibrational band is a unique success and gives a grand support to the DF model.

Table 2

B(E2) branching ratios. Calculated values which have hindrence/enhancement factor more than five are underline

			B(E2) branching ratios					
			$3^+ \to 2^+ / 4^{+'}$		$3^+ \to 2^+ / 2^{+'}$		$3^+ \to 4^+ / 2^{+'}$	
Nucleus	s	γ	Exp.	DF	Exp.	DF	Exp.	DF
1	2	3	4	5	6	7	8	9
^{74}Ge	2,021	29	0,0144	0,07215	0,01951	0,039	1,3476	0,5405
^{76}Ge	2,175	26,75	–	0,0846	0,0299	0,0424	–	0,5011
^{78}Se	2,132	27	–	0,0831	0,03197	0,042	–	0,5053
^{80}Se	2,175	26,75	–	0,0846	0,0366	0,0424	–	0,5011
^{98}Mo	2,233	26,25	0,2531	0,0876	–	0,0428	–	0,4884
^{100}Mo	2,731	23,5	–	0,1089	–	0,0525	–	0,4820
^{98}Ru	2,169	26,75	–	0,0846	–	0,0424	0,9387	0,5011
^{100}Ru	2,524	24,3	0,2763	0,113	0,0578	0,048	0,2092	0,4247
^{102}Ru	2,322	25,5	0,1473	0,0926	0,03721	0,0435	0,2525	0,4696
^{104}Ru	2,494	24,5	0,2650	0,1071	0,0376	0,047	0,1421	0,4388
^{102}Pd	2,757	23,5	–	0,1089	0,2508	0,0525	–	0,4820
^{104}Pd	2,414	25,0	0,0917	0,0962	0,0331	0,044	0,3616	0,4571
^{106}Pd	2,204	26,5	0,0788	0,0861	0,02612	0,0425	0,8308	0,4936
^{108}Pd	2,146	27,0	–	0,0831	0,0205	0,042	–	0,5054
^{110}Pd	2,176	26,75	–	0,0846	0,0244	0,0424	–	0,5011
^{106}Cd	2,714	23,5	–	0,1089	–	0,0525	–	0,4820
^{108}Cd	2,539	24,3	0,01795	0,113	–	0,048	–	0,4247
^{110}Cd	2,244	26,0	0,05649	0,0892	0,04065	0,043	0,7195	0,4820
^{112}Cd	2,134	27,0	–	0,0831	–	0,042	–	0,5054
^{114}Cd	2,166	26,75	–	0,0846	–	0,0424	–	0,5011
^{116}Cd	2,371	25,5	–	0,0927	–	0,0435	–	0,4692
^{122}Te	2,229	26,2	–	0,0876	0,01271	0,0428	–	0,4885
^{124}Te	2,200	26,5	–	0,0861	–	0,0425	–	0,4936
^{126}Te	2,132	27,0	–	0,0831	–	0,042	–	0,5054
^{150}Nd	8,165	13,8	–	0,7082	–	0,044	–	0,0621
^{146}Sm	2.2058	26,5	–	0,0860	0,181	0,0423	–	0,4941
^{148}Sm	2,642	23,7	–	0,1134	–	0,051	–	0,4497
^{150}Sm	3,575	20,5	0,3726	0,2615	0,0590	0,069	0,1583	0,2638
^{152}Sm	8,90	13,25	0,9498	0,6927	–	0,039	–	0,0563
^{154}Sm	17,55	9,5	0,9305	0,4	–	–	–	
^{152}Gd	3,22	21,5	0,4386	0,2045	0,03537	0,063	0,0806	0,3080
^{154}Gd	8,09	14,0	0,9701	0,716	0,06045	0,0463	0,06231	0,06466
^{156}Gd	12,97	11,0	0,2985	0,558	–	0,0172	–	0,03082

			$3^+ \rightarrow 2^+ / 4^{+'}$		B(E2) branching ratios $3^+ \rightarrow 2^+ / 2^{+'}$		$3^+ \rightarrow 4^+ / 2^{+'}$	
Nucleus	s	γ	Exp.	DF	Exp.	DF	Exp.	DF
1	2	3	4	5	6	7	8	9
^{158}Gd	14,93	10,3	2,6706	0,4305	–	0,0099	–	0,02297
^{158}Dy	9,56	12,7	1,4162	0,673	–	0,0341	–	0,05066
^{160}Dy	11,13	11,7	1,3555	0,618	–	0,0244	–	0,03948
^{162}Dy	11,01	12,0	1,6187	0,638	–	0,0269	–	0,04216
^{164}Dy	3,1439	21,75	–	0,1927	–	0,061	–	0,3165
^{156}Er	2,70	23,5	0,1828	0,1089	0,02845	0,0525	0,1555	0,4820
^{158}Er	4,26	18,75	0,5815	0,3392	0,04591	0,068	0,07895	0,20047
^{160}Er	6,80	15,0	0,6395	0,730	–	0,056	–	0,0767
^{162}Er	8,82	13,2	0,6986	0,6927	–	0,039	–	0,05630
^{164}Er	9,41	12,9	1,1308	0,683	–	0,0355	–	0,05197
^{166}Er	9,75	12,75	1,8991	0,673	0,01085	0,0341	0,00576	0,05066
^{168}Er	10,29	11,3	1,5765	0,578	–	0,0196	–	0,0339
^{170}Er	11,86	11,4	1,9181	0,596	–	0,020	–	0,0355
^{164}Yb	7,01	14,85	1,2528	0,7285	–	0,0547	–	0,07508
^{166}Yb	9,11	13,0	1,2106	0,685	–	0,0366	–	0,0534
^{168}Yb	11,21	11,7	1,5708	0,618	–	0,0244	–	0,03948
^{170}Yb	13,51	10,8	1,3223	0,535	–	0,0150	–	0,0280
^{172}Yb	18,61	9,5	1,9220	<0,4	–	–	–	–
^{176}Yb	15,35	10,25	–	0,4304	–	0,0999	–	0,2320
^{174}Hf	13,48	10,8	1,6151	0,535	–	0,0150	–	0,0280
^{176}Hf	15,18	10,25	1,4149	0,4305	–	0,0999	–	0,2320
^{178}Hf	12,61	11,3	–	0,578	–	0,0196	–	0,0339
^{180}Hf	13,93	10,7	–	0,511	–	0,0147	–	0,02876
^{182}W	12,20	11,4	2,0225	0,588	–	0,020	–	0,0340
^{184}W	8,12	13,8	1,5232	0,7082	–	0,044	–	0,06212
^{186}W	6,03	15,8	0,8893	0,588	–	0,0584	–	0,0993
^{186}Os	5,59	16,5	1,2817	0,489	0,070	0,060	0,0546	0,1226
^{188}Os	4,08	19,2	0,7239	0,3197	–	0,0697	–	0,2180
^{190}Os	2,99	22,5	0,3970	0,1404	0,07737	0,0575	0,3927	0,4095
^{192}Os	2,38	25,2	–	0,0944	0,08539	0,0437	–	0,4629
^{230}Th	14,69	10,5	3,4557	0,473	–	0,0123	–	0,0260
^{234}U	21,11	8,7	1,5296	<0,4	–	–	–	–
^{238}U	23,60	8,3	1,4659	<0,4	–	–	–	–
^{246}Cm	26,24	7,8	1,5516	<0,4	–	–	–	–

Table 3

$B(E2;\, 3^+ \rightarrow 2^+/2^{+\prime})$ values for $^{146-152}Sm$ nuclei.
Deviation of more than factor of 5 are underlined.
Experimental, DPPQ and BEM values are taken from reference 14

	$B(E2;\, 3^+ \rightarrow 2^+/2^{+\prime})$			
Nucleus	Exp.	DF	DPPQ	BEM
^{146}Sm	0,181	0,0425	0,100	–
^{148}Sm	–	0,051	0,066	–
^{150}Sm	0,059	0,069	0,285	<u>0,555</u>
^{152}Sm	–	0,039	38,461	2,5

Table 4

$B(E2;\, 3^+ \rightarrow 2^+/4^+)$ values for $^{98-104}Ru$ and $^{102-110}Pd$ nuclei.
Week's values are taken from reference 12

	$B(E2;\, 3^+ \rightarrow 2^+/4^+)$		
Nucleus	Exp.	DF	Weeks
^{98}Ru	—	0,0846	0,121
^{100}Ru	0,27633	0,1130	0,169
^{102}Ru	0,1473	0,0926	0,164
^{104}Ru	0,2650	0,1071	0,307
^{102}Pd	—	0,1089	0,47
^{104}Pd	0,09173	0,0962	0,271
^{106}Pd	0,0788	0,0861	0,173
^{108}Pd	—	0,0831	0,113
^{110}Pd		0,0846	0,286

Table 5

Non–axiality parameter values γ_1 and γ_2 calculated from $E2^{+\prime}/E2^+$ and $E3^+/E2^+$ energy ratios respectively

Nucleus	γ from $E2^{+\prime}/E2^+$ (γ_1)	γ from $E3^+/E2^+$ (γ_2)
^{74}Ge	29,0	–
^{76}Ge	26,75	–
^{78}Se	27,0	29,0
^{80}Se	26,75	
^{98}Mo	26,25	
^{100}Mo	23,5	
^{98}Ru	26,75	27,8
^{100}Ru	24,3	24,6
^{102}Ru	25,5	26,5
^{104}Ru	24,5	24,6
^{102}Pd	23,5	23,1
^{104}Pd	25,0	26,0
^{106}Pd	26.5	28,2
^{108}Pd	27,0	28,0
^{110}Pd	26,75	26,25
^{106}Cd	23,5	–
^{108}Cd	24,3	24,25
^{110}Cd	26,0	26,0
^{112}Cd	27,0	25,6
^{114}Cd	26,75	–
^{116}Cd	25,5	–
^{122}Te	26,2	24,75
^{124}Te	26,5	–
^{126}Te	27,0	–
^{134}Ba	30,0	–

Nucleus	γ from $E2^{+\prime}/E2^+$ (γ_1)	γ from $E3^+/E2^+$ (γ_2)
^{150}Nd	13,8	–
^{146}Sm	26,5	28,5
^{148}Sm	23,7	24,75
^{150}Sm	20,5	20,5
^{152}Sm	13,25	13,20
^{154}Sm	9,5	9,5
^{152}Gd	21,5	21,75
^{154}Gd	14,0	13,9
^{156}Gd	11,0	11,2
^{158}Gd	10,3	10,4
^{160}Gd	11,0	11,0
^{158}Dy	12,7	12,5
^{160}Dy	11,7	11,9
^{162}Dy	12,0	12,0
^{164}Dy	21,75	25,0
^{156}Er	23,5	23,5
^{158}Er	18,75	18,5
^{160}Er	15,0	15,0
^{162}Er	13,2	13,3
^{164}Er	12,9	13,0
^{166}Er	12,75	12,4
^{168}Er	11,3	12,2
^{170}Er	11,4	11,7
^{164}Yb	14,85	14,7
^{166}Yb	13,0	13,2

Nucleus	γ from $E2^{+'}/E2^{+}$ (γ_1)	γ from $E3^{+}/E2^{+}$ (γ_2)
^{168}Yb	11,7	11,9
^{170}Yb	10,8	10,9
^{172}Yb	9,5	9,25
^{174}Yb	9,0	8,8
^{176}Yb	10,25	10,25
^{174}Hf	10,8	10,8
^{176}Hf	10,25	10,25
^{178}Hf	11,3	11,5
^{180}Hf	10,7	10,8
^{182}W	11,4	11,4
^{184}W	13,8	13,9
^{186}W	15,8	15,8
^{186}Os	16,5	16,5
^{188}Os	19,2	19,2
^{190}Os	22,5	22,0
^{192}Os	25,2	25,5
^{230}Th	10,5	10,9
^{232}Th	10,0	–
^{234}U	8,7	8,7
^{236}U	8,7	8,75
^{238}U	8,3	8,3
^{240}Pu	8,6	–
^{242}Pu	8,15	–
^{244}Pu	8,58	–
^{246}Cm	7,8	7,8
^{248}Cm	8,0	–

Table 6

Calculated DF B(E2) branching ratios using parameters γ_1 and γ_2 for Ru and Pd chains of isotopes

	$B(E2; 3^+ \to 2^+/4^+)$			$B(E2; 3^+ \to 2^+/2^{+'})$			$B(E2; 3^+ \to 4^+/2^{+'})$		
Nucleus	Exp.	Calc. from (γ_1)	Calc. from (γ_2)	Exp.	Calc. from (γ_1)	Calc. from (γ_2)	Exp.	Calc. from (γ_1)	Calc. from (γ_2)
^{98}Ru	–	0,0846	0,0786	–	0,0424	0,041	0,9387	0,5011	0,521
^{100}Ru	0,2763	0,113	0,106	0,05781	0,048	0,046	0,2092	0,4247	0,433
^{102}Ru	0,1473	0,0926	0,086	0,03721	0,0435	0,0425	0,2525	0,4696	0,494
^{104}Ru	0,2650	0,1071	0,106	0,03766	0,047	0,046	0,1421	0,4388	0,433
^{102}Pd	–	0,1089	0,100	0,02508	0,0525	0,054	–	0,4820	0,540
^{104}Pd	0,09173	0,0962	0,089	0,03317	0,044	0,043	0,3616	0,4571	0,483
^{106}Pd	0,0788	0,0861	0,076	0,026123	0,0425	0,0405	0,8308	0,4936	0,533
^{108}Pd	–	0,0831	0,077	0,02054	0,042	0,041	–	0,5054	0,532
^{110}Pd	–	0,0846	0,0877	0,02446	0,0424	0,0428	–	0,5011	0,488

Table 7

Theoretical and experimental energies of $E3^+$ level

Nucleus	$E2^+$ Exp.	$E2^{+'}$ Exp.	$E3^+$ Theor.	$E3^+$ Exp.
1	2	3	4	5
^{74}Ge	0,59588	1,20431	1,80019	1,69722
^{76}Ge	0,56292	1,10845	1,67137	(1,5394)
^{78}Se	0,6134	1,3084	1,9218	(1,8536)
^{80}Se	0,66633	1,4495	2,11583	–
^{98}Mo	0,78742	1,7585	2,54592	–
^{100}Mo	0,5356	(1,463)	1,9986	–
^{98}Ru	0,65241	1,4149	2,06731	2,014
^{100}Ru	0,53959	1,3621	1,90169	(1,8812)
^{102}Ru	0,47507	1,10313	1,5782	1,52166
^{104}Ru	0,35799	0,8931	1,25109	1,2424
^{102}Pd	0,55641	1,53435	2,09076	2,1121
^{104}Pd	0,55579	1,34168	1,89797	1,82065
^{106}Pd	0,511862	1,12802	1,639882	1,5577
^{108}Pd	0,43395	0,93109	1,36504	1,3356
^{110}Pd	0,3738	0,8136	1,1874	1,2124
^{106}Cd	0,63269	1,7169	2,34959	–
^{108}Cd	0,63292	1,6070	2,23992	2,2395
^{110}Cd	0,657751	1,475774	2,133475	2,162763
^{112}Cd	0,61794	1,3123	1,93024	2,0641
^{114}Cd	0,55829	1,20928	1,76757	–
^{116}Cd	0,5139	1,2136	1,7215	–
^{122}Te	0,5640	1,25699	1,82099	(1,9406)
^{124}Te	0,60242	1,32550	1,92792	–
^{126}Te	0,66633	1,42017	2,0865	–
^{134}Ba	0,60466	1,16790	1,77256	1,64339
^{150}Nd	0,13012	1,0624	1,19252	–
^{146}Sm	0,74724	1,64833	2,39557	(2,269)
^{148}Sm	0,5503	1,4543	2,0046	(1,9029)
^{150}Sm	0,33395	1,19381	1,52776	1,50453
^{152}Sm	0,121782	1,08589	1,207672	1,23387
^{154}Sm	0,08198	1,4404	1,52238	(1,5400)
^{152}Gd	0,344282	1,109183	1,453465	1,433975
^{154}Gd	0,123070	0,99628	1,11935	1,12782
^{156}Gd	0,088965	1,15410	1,243065	1,24800
^{158}Gd	0,079510	1,187097	1,266607	1,265475
^{160}Gd	0,07526	0,988	1,06326	1,058
^{158}Dy	0,09894	0,94627	1,04521	1,04452
^{160}Dy	0,086788	0,966152	1,05294	1,04909
^{162}Dy	0,080660	0,88822	0,96888	0,96300
^{164}Dy	0,24230	0,76178	1,00408	0,82817
^{156}Er	0,3445	0,9304	1,2749	1,2430

Nucleus	$E2^+$ Exp.	$E2^{+'}$ Exp.	$E3^+$ Theor.	$E3^+$ Exp.
1	2	3	4	5
^{158}Er	0,19218	0,82013	1,01231	0,04341
^{160}Er	0,12562	0,85470	0,98031	0,98731
^{162}Er	0,10208	0,90068	1,00276	1,00192
^{164}Er	0,09139	0,86031	0,9517	0,94635
^{166}Er	0,080574	0,78589	0,866464	0,85938
^{168}Er	0,079804	0,821166	0,90097	0,895792
^{170}Er	0,07859	0,932	1,01059	1.0105
^{164}Yb	0,1238	0,8639	0,9877	1,0042
^{166}Yb	0,10238	0,93239	1,03477	1,03924
^{168}Yb	0,08773	0,9838	1,07153	(1,0669)
^{170}Yb	0,084262	1,13857	1,222832	1,22538
^{172}Yb	0,078750	1,46586	1,54461	1,54906
^{174}Yb	0,076480	1,6337	1,71018	1,7091
^{176}Yb	0,08213	(1,2609)	1,34303	(1,336)
^{174}Hf	0,09101	1,22681	1,31782	1,33665
^{176}Hf	0,08835	1,3413	1,42965	1,4458
^{178}Hf	0,093170	1,17464	1,26781	1,26886
^{180}Hf	0,093324	1,30036	1,393684	1,38157

Nucleus	$E2^+$ Exp.	$E2^{+'}$ Exp.	$E3^+$ Theor.	$E3^+$ Exp.
1	2	3	4	5
^{182}W	0,100106	1,22143	1,321536	1,33116
^{184}W	0,111207	0,903283	1,01449	1,005968
^{186}W	0,12230	0,73754	0,85984	(0,86178)
^{186}Os	0,13716	0,76750	0,90466	0,91048
^{188}Os	0,15503	0,63312	0,78815	0,79002
^{190}Os	0,18668	0,557978	0,744658	0,756028
^{192}Os	0,205774	0,489038	0,694812	0,690335
^{230}Th	0,05320	0,78139	0,83459	0,8258
^{232}Th	0,049369	0,7852	0,834569	—
^{234}U	0,04348	0,92671	0,97019	0,9691
^{236}U	0,045242	(0,9581)	1,003342	(1,0014)
^{238}U	0,044915	1,0603	1,105215	(1,1056)
^{240}Pu	0,042825	(0,93807)	0,980895	
^{242}Pu	0,04454	1,102	1,14654	—
^{244}Pu	0,046	(1,015)	1,061	
^{246}Cm	0,042852	1,12426	1,167112	1,16547
^{248}Cm	0,04340	(1,050)	(1,0934)	—

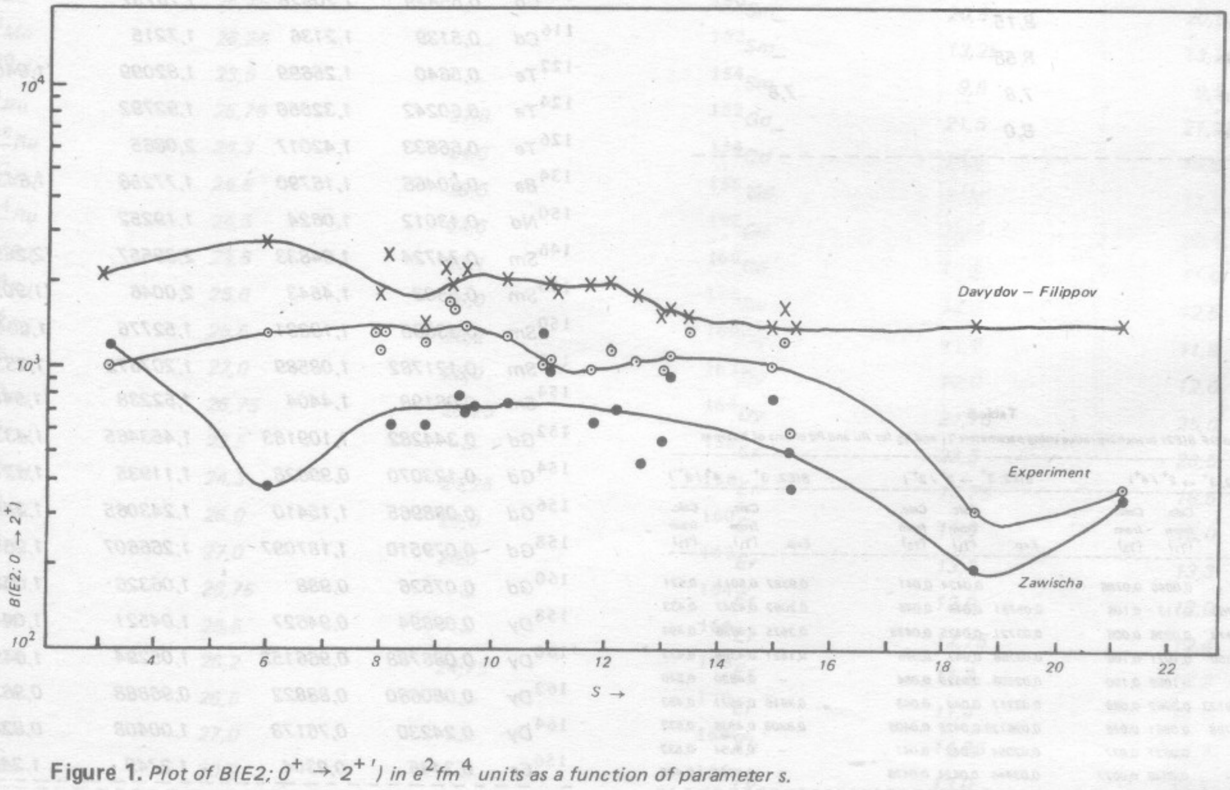

Figure 1. *Plot of $B(E2; 0^+ \rightarrow 2^{+'})$ in $e^2 fm^4$ units as a function of parameter s.*

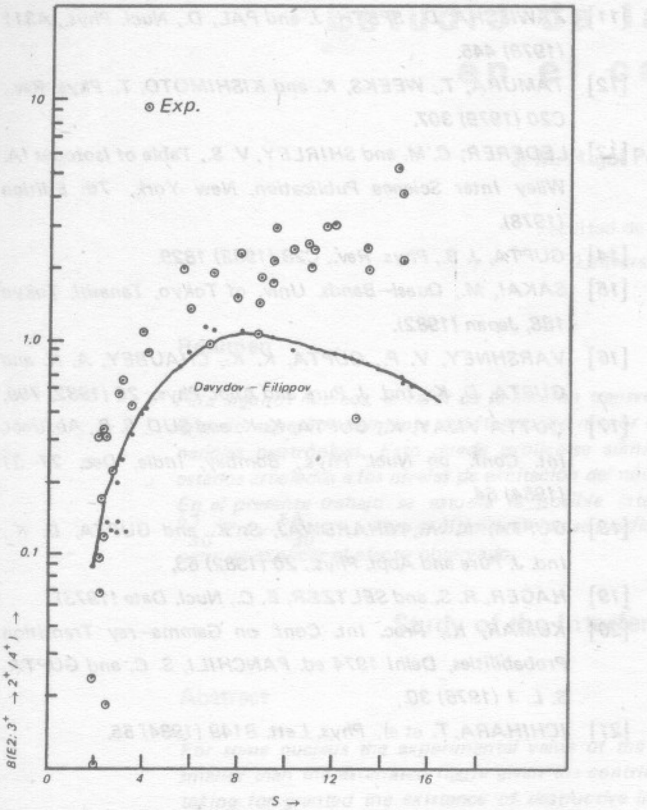

Figure 2. *Plot of $B(E2;\,3^+ \to\ 2^+ / 4^+)$ as a function of parameter s.*

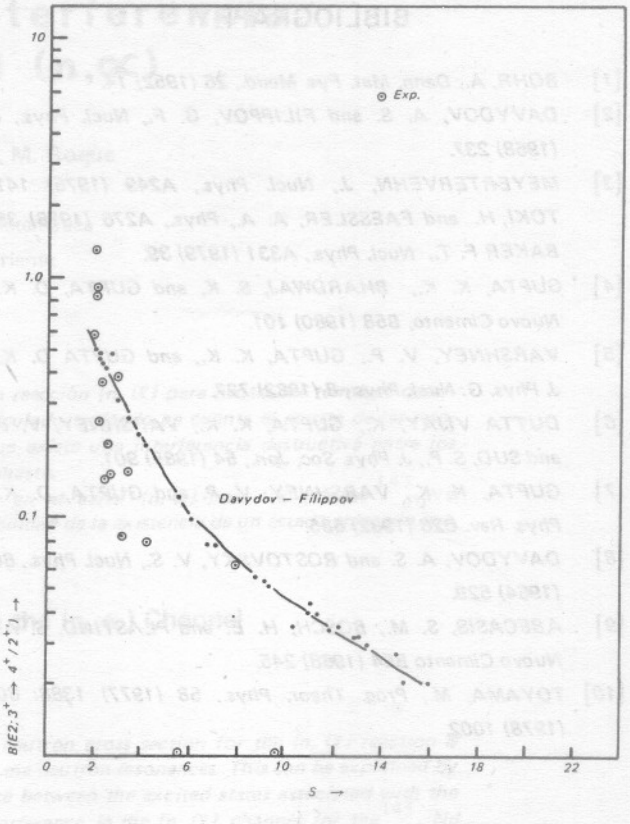

Figure 4. *Plot of $B(E2;\,3^+ \to\ 4^+ / 2^{+'})$ as a function of parameter s.*

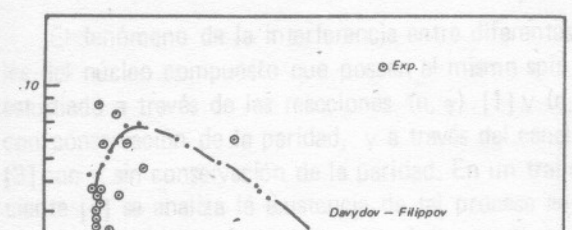

Figure 3. *Plot of $B(E2;\,3^+ \to\ 2^+ / 2^{+'})$ as a function of parameter s.*

CONCLUSION

It is inferred that the assumption of rigid–triaxial shape with fixed shape parameters β and γ is an excellent approximation to the actual nuclear wave functions. The gamma band which is a long standing problem [21] to the researches so far, is generated from the rotation of triaxial rigid rotor. The present study supports that $K^{\pi} = 2^+$, I=3 levels with energy of about 1–2 MeV are components of the quadrupole shape oscillations in contrast to Zawischa *et al*, who doubted the collective nature of low–lying levels and suggested that only high–lying $K^{\pi} = 2^+$ resonances were classical gamma vibrations.

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